## The Ideal Gas Constant

OBJECTIVE: This experiment is designed to provide experience in gas handling methods and experimental insight into the relationships between pressure, volume, temperature and the number of moles of a gas. One goal of the lab is the experimental determination of the ideal gas constant R .

SKILLS: manipulation of gases; PVT relationships, standard deviation

EQUIPMENT: Gas buret

REFERENCE: Chemistry: The Molecular Nature of Matter, Jespersen et al., $7^{\text {th }}$ edition, Sections 10.1-6

SAFETY AND DISPOSAL: Hydrochloric acid is corrosive. If you should get it on your skin, IMMEDIATELY go to the sink and wash with plenty of water and soap. If the spill is serious have another student notify the instructor while you continue to wash. Disposal of all liquids in the lab sinks at the end of the lab is appropriate.

INTRODUCTION: Gases are characterized experimentally by measuring their pressure $(\mathrm{P})$, volume $(\mathrm{V})$, moles ( n ) and temperature (T). These four variables are related by the ideal gas law: $\mathrm{PV}=\mathrm{n} R$, where R is a constant that does not depend on $P, V, n$ or $T$. Solving the ideal gas law for $R$ gives $R=P V / n T$. In this experiment you will determine the value of R by measuring $\mathrm{P}, \mathrm{V}, \mathrm{n}$ and T .

You will generate hydrogen gas for this purpose by reacting magnesium with hydrochloric acid. When metals react with acids, the products are usually the salt of the metal plus hydrogen gas. For example, magnesium reacts with hydrochloric acid and $\mathrm{Mg}(s)+2 \mathrm{HCl}(a q) \rightarrow \mathrm{Mg} \mathrm{Cl}_{2}(a q)+\mathrm{H}_{2}(g)$. By measuring the mass of metal used, it is possible to determine the number of moles ( n ) of hydrogen gas produced in each case. The theoretical prediction is that the ideal gas constant will not be affected by which metal is used to generate the hydrogen gas.

The experimental set-up you will be using is shown in Figure 1, before the magnesium and HCl have reacted, and in Figure 2 after their reaction. It is easy to measure the temperature $(\mathrm{T})$ and volume $(\mathrm{V})$ of the hydrogen gas that is produced. Determination of the pressure, however, is a bit more complicated and involves careful attention to units. You will collect the hydrogen gas over water. Because water has an appreciable vapor pressure, the volume occupied by the hydrogen gas also contains gaseous water molecules. The total pressure of the gas is the sum of the pressures due to hydrogen and to water: $\mathrm{P}_{\text {gas }}=\mathrm{P}_{\text {hydrogen }}+\mathrm{P}_{\text {water vapor }}$. The water vapor pressure $\mathrm{P}_{\text {water vapor depends }}$ upon its temperature as shown in Figure 3. To determine $\mathrm{P}_{\text {hydrogen }}$ you will have to correct the total gas pressure $\mathrm{P}_{\text {gas }}$ for the presence of water vapor.


Figure 1. Gas buret filled with HCl and water before inversion and reaction. Mg is tied to a string and held in place by a stopper.


Figure 2. Gas apparatus after inversion and reaction.

A diagram of the experimental system shown in Figure 2 is useful in finding $\mathrm{P}_{\text {gas }}$. There is a vertical tube which is closed at the top and which contains both hydrogen gas and water vapor. The tube is immersed in a beaker of water, and some of the water has risen part way into the tube. There are several different pressures involved. Atmospheric pressure $\mathrm{P}_{\mathrm{atm}}$ (from the air) pushes down onto the water on the surface of the beaker. Gas pressure $\mathrm{P}_{\mathrm{gas}}$ (from the hydrogen and water gases in the tube) pushes down on the column of water in the tube. In addition, there is a pressure on the water surface due to the weight of the water column in the tube above the level of the water in the beaker. This pressure, which increases with the height difference between the two levels, is $\mathrm{P}_{\text {height difference. }}$. Because of the way fluids behave, $\mathrm{P}_{\mathrm{atm}}$ at the level of the water on the surface of the beaker is equal to the pressure in the tube at this level. Thus

$$
\begin{equation*}
\mathrm{P}_{\mathrm{atm}}=\mathrm{P}_{\text {gas }}+\mathrm{P}_{\text {height difference }}=\mathrm{P}_{\text {hydrogen }}+\mathrm{P}_{\text {water vapor }}+\mathrm{P}_{\text {height difference }} \tag{1}
\end{equation*}
$$

In this experiment, you will use a barometer to measure the atmospheric pressure $\mathrm{P}_{\text {atm }}$ and use a ruler to measure the height difference between the water in the beaker and the water in the tube as shown in Figure 2. $\mathrm{P}_{\text {height difference }}$ depends on the mass of the water in the column. Because the density of Hg is 13.6 times the density of water, a column of water would be 13.6 times higher than a column of Hg of the same weight. Thus the pressure that is due to 1 mm Hg is equal to the pressure due to 13.6 mm of water. $(13.6 \mathrm{~mm}$ water $=1 \mathrm{~mm} \mathrm{Hg}=1$ torr)

$$
\begin{equation*}
\mathrm{P}_{\text {height difference }}(\mathrm{torr})=\mathrm{h}_{\mathrm{mm} \text { water }} / 13.6 \tag{2}
\end{equation*}
$$

Combining these expressions gives an overall equation for the pressure of hydrogen

$$
\begin{equation*}
P_{\text {hydrogen }}=P_{\text {atm }}-P_{\text {height difference }}-P_{\text {water vapor }} \tag{3}
\end{equation*}
$$

$P_{\text {atm }}$ is to be measured experimentally, while $P_{\text {height difference }}$ can be calculated from Eq. (2) using the height difference measured in millimeters. $P_{\text {water vapor }}$ can be found from the equation in Figure 3 for the experimental system temperature in degrees Celsius. It is helpful to do these calculations in mmHg and then convert $\mathrm{P}_{\text {hydrogen }}$ to atmospheres when calculating the value for R .


Figure 3. Water Vapor Pressure $(\mathrm{mmHg})$ as a function of Temperature $\left({ }^{\circ} \mathrm{C}\right)$. The equation given relates the temperature in ${ }^{\circ} \mathrm{C}$ to the vapor pressure of water in mmHg .

When a measurement is repeated several times it is useful to determine how much variation there is between each trial. The standard deviation (usually given the symbol $\sigma$ ), measures on average how much each measurement differs from the mean or average value. If many data points are close to the mean and the scatter in the data is small, then the standard deviation is small. Conversely, if many data points are far from the mean and there is a large amount of scatter in the data, then the standard deviation is large. If all data values are equal, then the standard deviation is zero. Standard deviation is a measure of precision. To calculate the standard deviation the following formula can be employed:

$$
\sigma=\sqrt{\frac{1}{N} \sum_{i=1}^{N}\left(x_{i}-\bar{x}\right)^{2}}
$$

Where $N$ is the total number of measurements made, $x_{i}$ is a particular measurement, $\bar{x}$ is the mean or average value of all data collected. Excel also has a standard deviation function built in, called STDEV.

## EXPERIMENTAL PROCEDURE

Obtain a 50 ml gas buret, a one-hole " 00 " rubber stopper and a 600 ml beaker. Place approximately 450 ml of distilled water in the 600 ml beaker. Your instructor will demonstrate the set-up of the equipment.

Polish a small strip of magnesium with some steel wool. Next, obtain a piece of magnesium metal weighing between 0.040 g and 0.045 g . Record the mass of the metal. Wrap the metal with string so that it will hang down in the gas buret. Your instructor will demonstrate how to do this. Place the gas buret in a buret holder on a ring stand. Add 10 mL of 6 M HCl (Hydrochloric acid) to the gas buret using the dispensing station located in the fume hood. Using your wash bottle, carefully add distilled water down the side of the buret, without disturbing the hydrochloric acid you just placed in the buret. If done properly, the distilled water will remain "on top of" the hydrochloric acid and not mix with it. Continue adding water until the buret is completely filled.

Holding onto the string, carefully place the metal so that it hangs several centimeters down into the gas buret as shown in Figure 1. Gently insert the stopper into the buret so that the stopper holds the string in place. Check to be sure that an air bubble has not formed at the stopper. If an air bubble is present, remove the stopper, add more distilled water and re-insert the stopper. Using your distilled water bottle, fill the holes of the stopper with distilled water. Invert the gas buret into the beaker so that the top of the buret is below the water level in the beaker. Clamp the buret in place and allow the reaction to proceed. The more dense hydrochloric acid solution will diffuse downward and react with the metal to produce hydrogen gas. Note your observations which show that a reaction is occurring.

Your instructor will demonstrate how to use the laboratory barometer and write the day's atmospheric pressure on the board.

When the reaction is complete, read the volume of gas in the buret to the nearest $\mathbf{+ 0 . 0 2} \mathbf{m L}$. Then measure the height difference between the water in the beaker and the solution in the buret, as shown in Figure 2. This can be done in millimeters using a lab ruler. Measure the temperature of the solution in the beaker, which is equal to the temperature of the gas.

Rinse out your gas buret and do two more trials using magnesium.

Notebook Template: The data you will need for each trial in this experiment includes (recorded units are given):

- mass of magnesium metal used (grams)
- volume of hydrogen collected (liters)
- measured height difference between water levels in the beaker and gas buret $\left(\mathrm{mmH}_{2} \mathrm{O}\right)$
- temperature (Celsius and Kelvin)
- atmospheric pressure (inHg)
- vapor pressure of water at the experimental temperature ( mmHg ) [This can be obtained using the equation given in Figure 3, be careful to use the correct units as described in the figure.]


## LAB REPORT

The lab report for this experiment will include using your data to determine the value of the ideal gas constant R for hydrogen gas generated using magnesium. You will also need to calculate the standard deviation of your three data points. The formula for standard deviation is given in the introduction and the following steps can be used to determine value of $\sigma$ for your data:

1. Find the mean or average value $\bar{x}$
2. For each calculated value of R (each data point $\left.x_{i}\right)$ calculate the deviation from the mean $\left(x_{i}-\bar{x}\right)$
3. Calculate the square of each of the deviations: $\left(x_{i}-\bar{x}\right)^{2}$
4. Find the average or mean of the square deviations. This can be done by adding up each value from step 3 and dividing by the total number of measurements, which in this case is 3 . This quantity is referred to as the variance or $\sigma^{2}$.
5. To find the standard deviation, take the square root of the variance calculated in step $4: \sqrt{\sigma^{2}}$

In addition to working with your own data, you will use Excel to calculate the value of R for each data set measured in the class and determine class values of R . You will compare your results and the class averages to the known value of $\mathrm{R}=0.08206 \mathrm{~L} \mathrm{~atm} / \mathrm{mol}$ deg. Attention to significant figures will help in interpreting any differences.

In your calculation of class values for R, the downloaded Excel file will contain the mass of metal, volume of gas collected, temperature, height difference and atmospheric pressure for each trial. Be mindful of the units used in each column for the recorded data. Using the class data, you will need to calculate gas pressure for each student trial. You found the pressure of $\mathrm{H}_{2}$ gas in atm for your own data, so you should use the same approach for the class data.

$$
\mathrm{P}_{\text {Hydrogen gas }}=\mathrm{P}_{\text {atmospheric }}-\mathrm{P}_{\mathrm{H} 2 \text { Ovapor }}-\mathrm{P}_{\text {Height Difference }}(\text { where all need to be in } \mathrm{mmHg}) \text {. }
$$

So for example, to calculate the pressure of hydrogen gas in row 10 , you might enter the following formula in cell I12:
$=($ atmospheric pressure in inHg*25.4-G12-H12/13.6)/760
where the first part atmospheric pressure in inchesHg $* 25.4$ is the atmospheric pressure converted to mmHg (atmospheric pressure in inchesHg is recorded in cell E5), G12 is the vapor pressure of water in mmHg and $\mathrm{H} 12 / 13.6$ is the pressure due to the height difference converted to mmHg . So, the terms inside the () will give you the pressure of hydrogen gas in mmHg . By dividing the pressure by 760 , you convert it to atm. The pressure of the hydrogen gas should be less than 1 atm .

You will need to calculate the other parameters and the value of R for each trial. Finally, the average value for R is needed. To calculate the average, you can go to an empty cell and use the Excel command "=average(". This will open a window that asks you to highlight the values you want averaged. When you press Enter, the average value will appear in that cell. To get the average R from the all trials, simply highlight the R values that you have calculated for them. Before averaging, inspect the calculated values of $R$ to see if any are obviously wrong and need to be discarded. If there are any, make a note of them for the report and type a space in the cell with the value to be discarded. (The average command will ignore any blank spaces, but will include any cell with a 0 in it.)

The standard deviation of the class data can be calculated in a similar manner to the class average. Go to an empty cell and use the Excel command "=STDEV(" This will open a window that asks you to highlight the values you want to know the standard deviation of. After highlighting all of the calculated values of R for the class, press Enter and the standard deviation of the class data will appear. You can obtain the same value by using the "Insert Function" command and selecting the standard deviation function.

You will also be asked to calculate the relative error, defined as the difference between the experimental and known values, relative to the known value, expressed as a percentage:

$$
\text { Relative error }=(\text { experimental value }- \text { known value }) /(\text { known value }) * 100 \% \text {. }
$$

A positive (negative) relative value means that the experimental result is higher (lower) than the known one. You will determine the relative errors for your data and for the class data.

